Magneto-Biofluid Flow of Lipoprotein through an Inclined Arterial Channel with Haematocrit

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Abstract:

Mathematical models were used in the investigation of mass transfer of magnetic drug on lipoprotein during MHD blood in an arterial channel. The non-linear partial differential equations were changed into linear partial differential equations and were then solved analytically to obtain the exact solutions. The numerical results were obtained through plots showing the parameter variations on their effects on the focus of the study. With the help of graphs parameters such as Grashof number, Schmidt number, magnetic parameter were determined. The significance of this study is on the therapeutic analysis of hyperthermia more especially on the regulation of blood flow and lipid in blood.

Keywords: *Magnetic drug targeting, magnetohydrodynamics(mhd), Lipoprotein.*

Introduction

Magnetohydrodynamic (MHD) fluid is a fluid that conducts electricity in electric and magnetic fields. It incorporates fluid dynamics and electromagnetic assertions to describe concurrent effects of magnetic field on the flow and vice versa. Its concern is on gases that are ionized and liquids that are electrically conducting. Varieties of papers have evolved over the years on this concept. Take for instance; Singh and Mathew(2008) studied the effects that injection/suction has on oscillating hydrodynamic magnetic flow in a horizontal channel that is rotating. Attia and Kotb(1996) examined magnetohydrodynamic flow between parallel plates having heat transfer. Swapna et al.(2017) studied mass transfer on mixed convective periodic flow through porous medium in an inclined channel. Achogo et al.(2020) examined magnetohydrodynamic convective periodic flow through a porous medium in an inclined channel with thermal radiation and chemical reaction.

The old pattern of drug targeting requires the injection of drugs into the stream of blood, which will be uniformly distributed in all the body. Nevertheless, when attending to a targeted or a definite area, this method of drug targeting is not required as enough will be needed for treating that required area (Bali et al., 2016). Magnetic drug targeting (MDT) is amongst others one of the way drug targeting procedures due to its splendid efficiency and minute toxic side effects on healthy cells and tissues (Alexiou et al., 2011). Furlani and Ng (2006) presented an analytic model for predicting the transport and capture of magnetic nanoparticles in the microvasculature. Furlani and Furlani (2007) have studied the mechanism of magnetic targeting of carrier particles in the microvasculature for therapeutic use. In magnetic drug targeting (MDT), various forces such as the magnetic force, fluidic force and buoyant force as the dominant force are acting on the multifunctional carrier particles (Sutradhar et al., 2016). In this direction, several investigators have studied to understand the influence of different parameters on the trajectories of the drug carriers by considering different rheology of blood (Mahmoodpour et al., 2020; Zafar et al., 2019; Sharma et al., 2015a, 2015b), permeability of the blood vessel (Shaw et al., 2017), external magnetic field (Rukshin et al., 2017; Choomphon-anomakhun et al., 2017), and two-phase blood flow model (Sibanda and Shaw, 2014).

The investigation of blood flow through arteries is considerably very important in many cardiovascular diseases. The poor circulation of blood in the body due to occlusion/blockage in arteries is one major health risks. Arteries channels oxygenated blood with nutrients from the heart to the tissues of the body, in the circulatory system of the human body. Blood is a viscous fluid circulating in the artery/vein. It has a strong nourishing effect on the human body and serves as one of the basic substances constituting the human body. Blood is a wonderful fluid which is an important factor of life. Atherosclerosis is hardening of a blood vessel from a buildup of plaque as a result of excessive cholesterol (lipoprotein) intake. Plaque is made of fatty deposits, cholesterol, and calcium. Plaque buildup causes the artery to narrow and harden which is a serious risk human living. Plaque buildup can slow and even stop blood flow. This means the tissue supplied by the artery is cut off from its blood supply and as such humans must watch the quantity of cholesterol they consume. It often leads to pain or decreased function. This condition can cause a number of serious health problems. Over the last two decades there has been theoretical and experimental studies of blood flow through the circulatory system of living mammals, has been the subject of scientific research and literatures available such as: characteristics of blood flow through an artery in the presence of multistenosis were studied by Chakravarty and Sannigrahi (1999) the investigation of basic BFD flow problems attracts interest due to the numerous proposed applications in bioengineering and medical sciences. Bio-fluids in the presence of a magnetic field with dissipation finds its applications in various upcoming fields like innovative drug targeting, surgical operations, etc. Haik et al. (2001) reported a 30 percent Investigation of Lipoprotein based MHD Fluid Flow Through an Arterial Channel with Haematocrit decrease in blood flow rate when subjected to a high magnetic field of 10 T while Yadav et al. (2008) found a similar reduction in blood flow rate but at a much smaller magnetic field of 0.002 T. Sharma et al. (2013) formulated a mathematical model for the hydro -magnetic bio -fluid flow in the porous medium with Joule effect. A theoretical analysis of blood flow and heat transfer in a permeable vessel in the presence of an external magnetic field was made by Sinha et al. (2016). Shit and Roy (2016) investigated the effect of induced magnetic field on blood flow through a constricted channel, and demonstrated that increasing the values of the magnetic field reduces the velocity of the blood flow at the center. Rahbari et al. (2017) carried

out an analytical study on blood flow containing nanoparticles through porous blood vessels in the presence of the magnetic field using the Homotopy Perturbation Method (HPM). Blood flow in a large blood vessel has a profound influence on the efficiency of thermal therapy treatment. Electromagnetic heat, such as short waves and microwaves, sends heat up to 2 inches into the tissue and muscles. It works best for injuries in joints, muscles, and tendons. Moreover, hyperthermia treatment is found to be effective during cancer therapy in recent years. Its objective is to raise the temperature of pathological tissues above cytotoxic temperatures (41 – 450C) without overexposing healthy tissues (1994). Heat and mass transfer of blood flow considering its pulsatile hydro -magnetic rheological nature under the presence of viscous dissipation, Joule heating and a finite heat source was discussed by Sharma et al. (2015). Sinha and Shit (2015) investigated the combined effects of thermal radiation and MHD heat transfer blood flow through a capillary. Thermal radiation effect on inclined arterial blood flow through a non -Darcian porous medium with magnetic field was discussed by Sharma et al. (2015). Bunonyo et al (2018) investigated blood flow a stenosed artery with heat in the presence of magnetic field. In their investigation it is observed that magnetic field increase inhibit blood flow as a result of Lorentz force.

Mathematical Formulation

We consider a blood flow by making the assumption that the artery is the channel, and blood as an incompressible Newtonian fluid, viscous and electrically conducting. The viscous nature of blood is taking to be the percentage of red blood and lipid. The lipoprotein is the protein in the blood which gives rise to deterrents to the flow with an increase in concentration. The heart pumps the blood causing the flow of the blood. As an addendum, we also make the assumption u to be the velocity of the fluid, $C_w \, and \, C_\infty$ as the concentration of lipoprotein at the wall and far field, D0 as the molecular diffusivity, S is the external lipid source and H is the hematocrit. The governing equation for the flow of the fluid through an artery is stated as coupled partial differential equations as stated below.

$$\rho \frac{\partial u\prime}{\partial t\prime} = -\frac{\partial p\prime}{\partial x\prime} + \frac{\partial}{\partial y\prime}\left(\mu'(H)\frac{\partial u\prime}{\partial y\prime}\right) + \rho g \beta_c (C' - C_\infty) Sin \propto + \sigma B_0^2 u' \quad (1)$$

$$\frac{\partial C\prime}{\partial t\prime} = \frac{\partial}{\partial y\prime}\left(D'\frac{\partial C\prime}{\partial y\prime}\right) + S(C') + Kr'(C' - C_\infty) \quad (2)$$

and the corresponding boundary conditions are thus as follows

$$u' = 0, \quad C' = C'_\infty \quad at \quad y' = 0 \quad (3a)$$

$$u' = 0, \quad C' = C'_w \quad at \quad y' = R_0 \quad (3b)$$

We assumed that the lipoprotein concentration dependent on the fluid viscosity, mass diffusion and external lipoprotein–C source respectively as:

$$\mu = \frac{\mu_0}{(1+2.5H)}, S = Q\left(b_3(C' - C_\infty)\right), D' = D_0$$
(4)

We non dimensioned the models using the following dimensionless quantities;

$$y = \frac{y'}{R_0}, x = \frac{x'}{R_0}, u = \frac{u'^{R_0}}{\nu}, u = \frac{\nu t'}{R_0^2}, \mu = \frac{\mu'}{\mu_0}, \phi = \frac{C' - C_\infty}{C_w - C_\infty}, P = \frac{P' R_0^2}{\rho \nu^2}, Gr = \frac{g R_0^3 \beta_c (C_w - C_\infty)}{\nu^2},$$

$$Sc = \frac{\nu}{D_0}, \nu = \frac{\mu_0}{\rho}, \lambda = \frac{Q b_3 R_0^2}{\nu}, Kr = \frac{Kr' R_0^2}{D_0}, -\frac{\partial P}{\partial x} = P e^{i\omega t}$$
(5)

Hence, utilizing equation (5) into equations(1) and (2) together with the corresponding boundary conditions, we obtain;

$$(1 + 2.5H)\frac{\partial u}{\partial t} = -(1 + 2.5H)\frac{\partial P}{\partial x} + \frac{\partial^2 u}{\partial y^2} + (1 + 2.5H)GcSin \propto \emptyset - (1 + 2.5H)M^2 u$$
(6)

$$\frac{\partial \emptyset}{\partial t} = \frac{1}{Sc}\frac{\partial^2 \emptyset}{\partial y^2} + \lambda \emptyset + \frac{Kr}{Sc}\phi$$
(7)

And the corresponding boundary conditions are also as follows;

u=0 , $\emptyset = 0$ at y=0
(8a)

u=0 , $\emptyset = 1$ at y=1
(8b)

Method of Solution

We at this point assumed the solution in terms of one term perturbation technique inorder to express equations (6)-(8) to depend on space only. The assume solutions as follow;

$$u(y,t) = u_0 e^{i\omega t} \quad (9a)$$

$$\phi(y,t) = \phi_0 e^{i\omega t}$$
(9b)

We now substituted equation (9) into equations (6)-(8) and we obtain;

$$\frac{d^2 u_0}{dy^2} - \mathrm{m}_1 u_0 = -L(P + GcSin \propto \phi_0)$$
(10)

$$\frac{d^2 \phi_0}{dy^2} + x_2 \phi_0 = 0 \quad (11)$$

where L=$1 + 2.5H$, x_2=$(\lambda - i\omega)Sc + Kr$ and m_1= $L(M^2 + i\omega)$

With the boundary condition as;

$u_0 = 0, \quad \phi_0 = 0$ at y=0 (12a)

$u_0 = 0, \quad \phi_0 = e^{-i\omega t}$ at y=h
(12b)

Again we assumed the general solution of equation (11) since it is an ordinary differential equation as;

$$\phi_0(y) = Asin\sqrt{x_2}y + Bcos\sqrt{x_2}y$$
(13)

Solving equation (13) together with the boundary condition equation (12)

we obtain;

B=0 and $A = \frac{e^{-i\omega t}}{Sin\sqrt{x_2}h}$
(14)

Hence;

$$\phi_0(y) = (\frac{e^{-i\omega t}}{sin\sqrt{x_2}h} sin\sqrt{x_2}y)\, e^{i\omega t}$$
(15)

and

$$\phi_0(y,t) = \frac{sin\sqrt{x_2}y}{sin\sqrt{x_2}h}$$
(16)

Now substituting equation (16) into equation (10)

$$\frac{d^2u_0}{dy^2} - m_1u_0 = -LP - L\, Gc(\frac{e^{-i\omega t}}{sin\sqrt{x_2}h} sin\sqrt{x_2}y)Sin \propto$$
(17)

Equation (17) is non homogeneous, we first obtain the complimentary solution as;

$$\frac{d^2u_0}{dy^2} - m_1u_0 = 0$$
(18)

We express the complimentary solution as;

$$u_{0c}(y) = A_1Sinh\sqrt{m_1}y + B_1Cosh\sqrt{m_1}y \quad (19)$$

And the particular solution as;

$$u_{0p}(y) = A_2 + A_3 Sin\sqrt{x_2}y + B_2 Cos\sqrt{x_2}y$$
(20)

With $A_2 = \frac{LP}{m_1}$, $A_3 = (\frac{e^{-i\omega t}}{sin\sqrt{x_2}h})\left(\frac{LGrSin\propto}{x_2+m_1}\right)$, $B_2 = 0$, $B_1 = -\frac{LP}{m_1}$, $A_1 = \frac{LP}{m_1 Sinh\sqrt{m_1}}(Cosh\sqrt{m_1} - 1) - (\frac{e^{-i\omega t}}{sinh\sqrt{m_1}h})\left(\frac{LGrSin\propto}{x_2+m_1}\right)$

The solution for the space dependent velocity is;

$$u_0(y) = \left(\frac{LP}{m_1}\left(\frac{(Cosh\sqrt{m_1}-1)}{Sinh\sqrt{m_1}}\right) - \left(\frac{e^{-i\omega t}}{sinh\sqrt{m_1}h}\right)\left(\frac{LGrSin\propto}{x_2+m_1}\right)\right) Sinh\sqrt{m_1}y + \frac{LP}{m_1}(1 - Cosh\sqrt{m_1}y) + \left(\frac{e^{-i\omega t}}{sinh\sqrt{x_2}h}\right)\left(\frac{LGrSin\propto}{x_2+m_1}\right) sin\sqrt{x_2}y$$
(21)

And the final solution for the momentum equation is;

$$u(y,t) = (\left(\frac{LP}{m_1}\left(\frac{(Cosh\sqrt{m_1}-1)}{Sinh\sqrt{m_1}}\right) - \left(\frac{e^{-i\omega t}}{sinh\sqrt{m_1}h}\right)\left(\frac{LGrSin\propto}{x_2+m_1}\right)\right) Sinh\sqrt{m_1}y + \frac{LP}{m_1}(1 - Cosh\sqrt{m_1}y) + \left(\frac{e^{-i\omega t}}{sinh\sqrt{x_2}h}\right)\left(\frac{LGrSin\propto}{x_2+m_1}\right) sin\sqrt{x_2}y)e^{i\omega t}$$
(22)

Result and Discussion

Figure 1 shows that as the Schmidt number is increased from 1 to 4 units; a decrease is noted in the flow concentration profiles. Figure 2 shows the influence of the chemical reaction on the flow concentration. It is depicted that increasing the chemical reaction causes an increase in the flow concentration. Figure3 is the impact of the level of lipoprotein on the concentration profile. Increasing the lipoprotein from a source from 0.2 to 0.4 units actually increases the level of the concentration flow. Figure 4 shows the effect of oscillation on the concentration profile. The frequency of oscillation is increased from 1 to 4 units. It is seen that increasing the oscillation increases the concentration flows. Figure 5 shows the height of stenosis. It is seen that increasing h from 3 to 12; a decrease is experienced in the flow concentration. It is depicted in figure 6 and 7 the impact of Schmidt number and chemical reaction parameter on the velocity of the blood. It is shown that increasing the Schmidt number and chemical reaction parameter slows down the velocity of the blood. The impact of the magnetic parameter is shown in figure 8. From figure 8; it is clearly shown that the magnetic field parameter deters the blood flow owing to the presence of Lorentz force in the field. Lorentz force opposes motion. Hence, a decrease is noted in the blood flow with increasing magnetic field parameter. Figure 9 shows the impact of Grashof number on the velocity of the blood. Increasing the Grashof number increases the blood flow. The result depicts that the concentration difference between the wall and far field is more than the dynamic viscosity of the blood. The influence of the height of the stenosis is noted in figure 10. From figure 10, it is seen that increasing h from 3 to 12 units significantly causes a decrease in the velocity of the blood. Figure 11 shows the effect of Haematrocrit on the velocity. Haematocrit is the percentage of red blood cells found in the blood that shows the presence of

Haemoglobin in the blood. Yet it is noted in figure 11 that increasing haemotocrit level from 1 to 4 units increases the velocity at some points while at some other points of the flow a decrease is noted with a rate of divergence due to increase in viscosity. Figure 12 shows the effect of the level of lipoprotein from source. It is shown that increase in cholesterol significantly increases the viscosity of blood because of its addition to the rate of protein value presence in the blood plasma leading to a drop in the flow and thereby causing the rate at which the heart beats to increase as shown in figure 12. Figure 13 shows the effect of the frequency of oscillation on the velocity of the blood. It is seen that increasing the frequency from 1 to 4 units detards the blood flow.

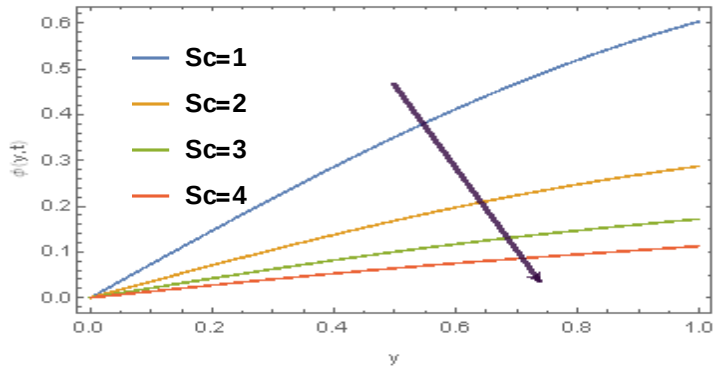

Fig1: Impact of Sc on concentration profile

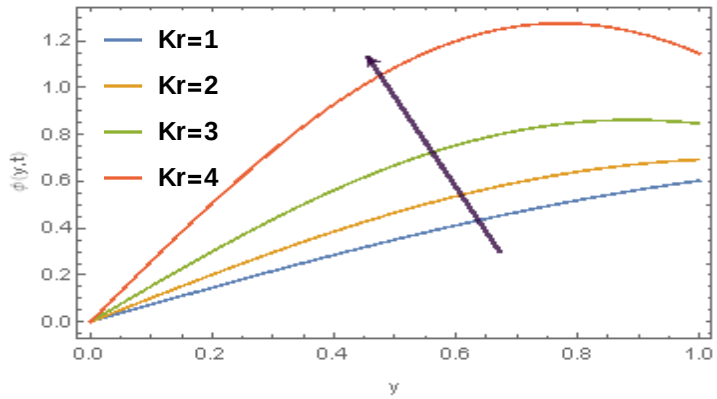

Fig2: Impact of Kr on concentration profile

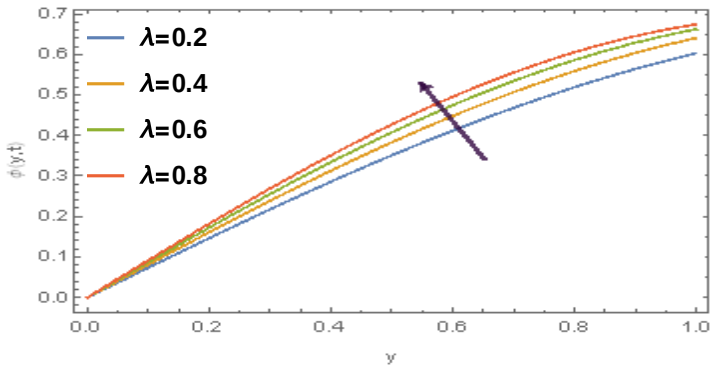

Fig3: Impact of λ on concentration profile

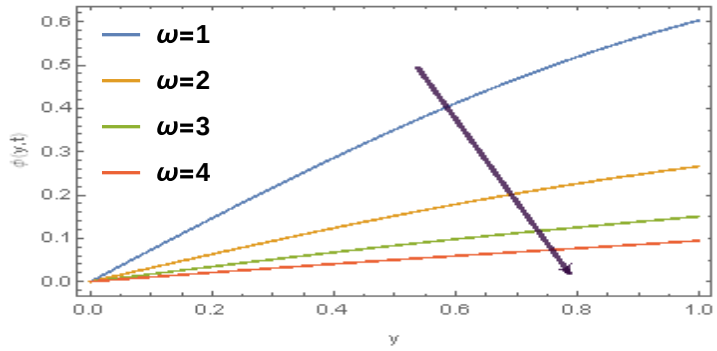

Fig4: Impact of ω on concentration profile

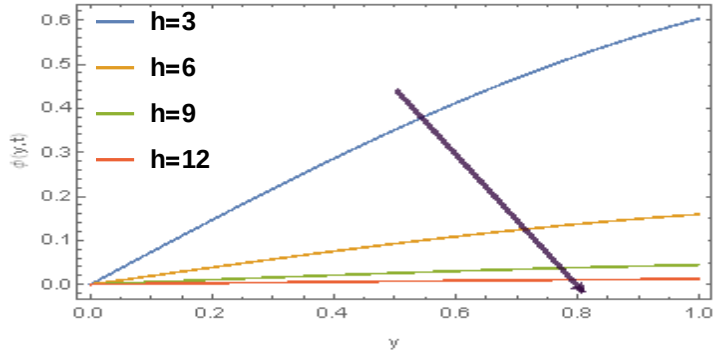

Fig5: Impact of h on concentration profile

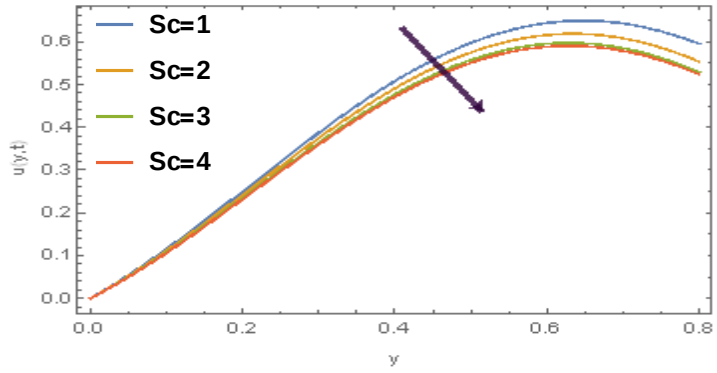

Fig6: Impact of Sc on momentum profile

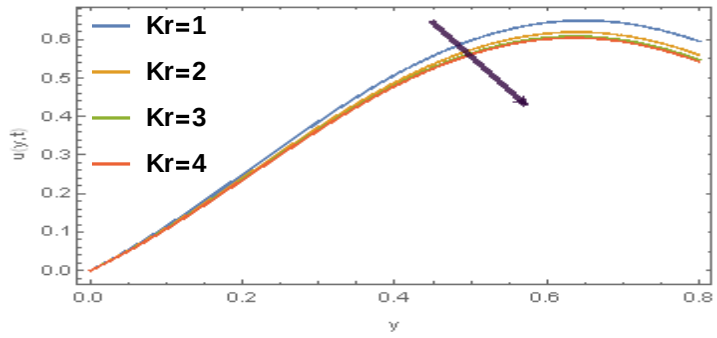

Fig7: Impact of Kr on momentum profile

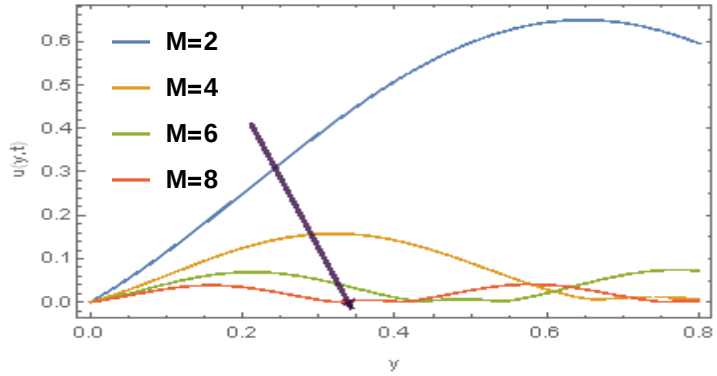

Fig8: Impact of M on momentum profile

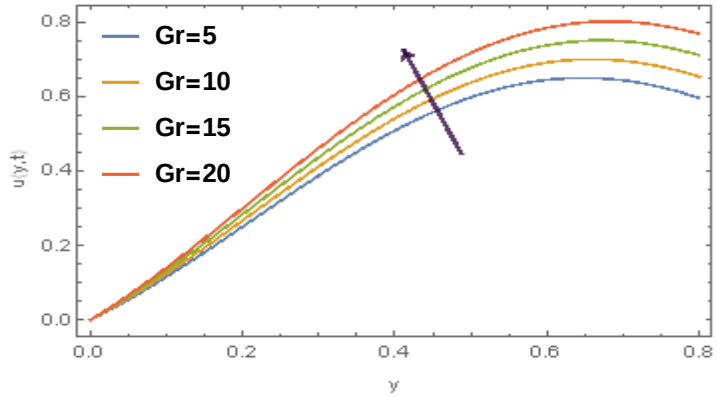

Fig9: Impact of Gr on momentum profile

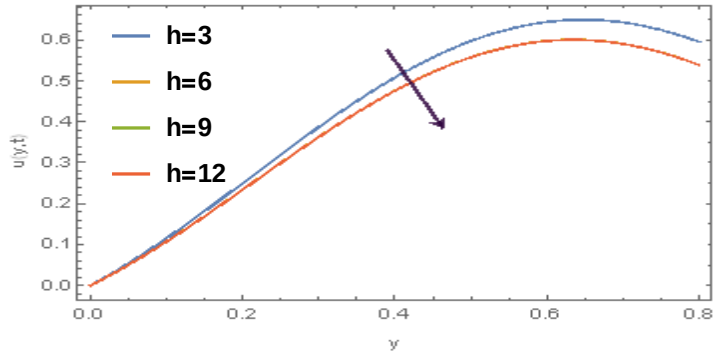

Fig10: Impact of h on momentum profile

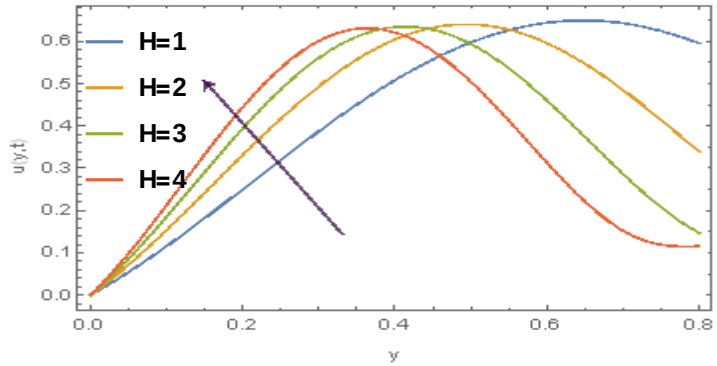

Fig11: Impact of H on momentum profile

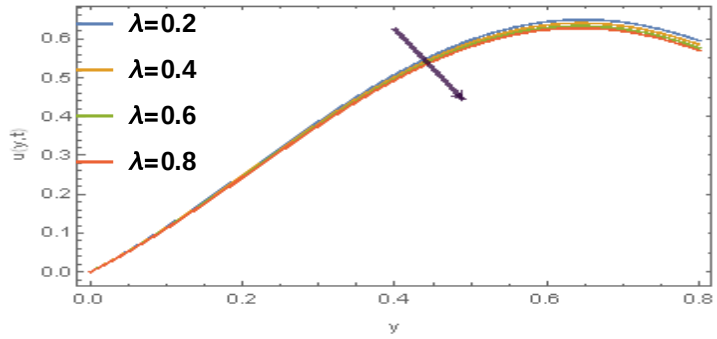

Fig12: Impact of λ on momentum profile

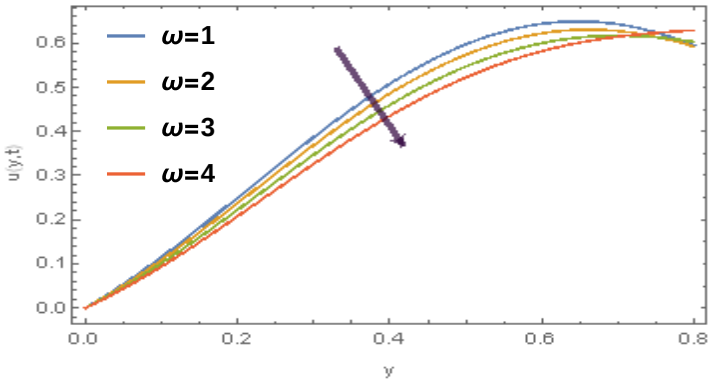

Fig13: Impact of ω on momentum profile

CONCLUSION

In this paper, we have studied the mass transfer of magnetic drug on lipoprotein during mhd blood in an arterial Channel**.** The governing equations were solved using analytical approach. From the study, the following have been drawn.

1. Increasing the level of lipoprotein causes a significant increase in the concentration flow.
2. Increasing Grashof number increases the velocity of the blood.
3. The magnetic parameter increases with a decrease in the velocity of the blood.
4. Increasing the frequency of oscillation causes the velocity of the blood to drop.

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